arXiv:0712.1478v1 [hep-ph] 10 Dec 2007

HEAVY QUARKONIA IN LIGHT-FRONT QCD

Stanisław D. Głazek

Institute of Theoretical Physics, University of Warsaw, Poland

Abstract

This talk is based on results obtained for masses and wave functions of heavy quarkonia in a light-front Hamiltonian formulation of QCD with just one flavor of quarks using an ansatz for the mass-gap for gluons. Since the calculated spectra compare reasonably well with data, some further steps one can make are discussed.

1 Introduction

Discussion of heavy quarkonium dynamics in this talk is based on results for masses and wave functions obtained in Refs. [1, 2] in a relativistic (boost-invariant) Hamiltonian formulation of QCD. Steps involved in the calculation, starting with the Lagrangian for QCD, deriving the corresponding canonical

light-front (LF) Hamiltonian, carrying out the renormalization group procedure for effective particles (RGPEP) to obtain the quark and gluon operators at finite momentum scales, λ, deriving an effective Hamiltonian for heavy constituent quarks and gluons, H_λ, at momentum scales on the order of the quark mass, $\lambda_0 \sim m$, using an ansatz for the mass-gap for gluons, μ, to finesse a new Hamiltonian that acts only in the effective quark-antiquark Fock sector, $H_{\lambda_0 Q\bar{Q}}$, and solving numerically the resulting eigenvalue problem for $H_{\lambda_0 Q\bar{Q}}$, are described in the original literature (a condensed summary [3] is available).

Here, only one example of results for quarkonium masses obtained from $H_{\lambda_0 Q\bar{Q}}$ is quoted, to illustrate what happens in the simplest version of the LF approach to QCD. The key point is that the results do not depend on the ansatz μ and fit data reasonably well for the coupling constant expected from RGPEP, assuming it has a known value at $\lambda = M_Z$, and for the charm or bottom quark masses that have typically considered sizes. Then, the emerging recipe for the mass gap ansatz as a tool to facilitate numerical studies of effective quark and gluon dynamics is described. The ansatz is designed to be introduced only in the final stage of diagonalizing $H_{\lambda_0 Q\bar{Q}}$.

LF quantum field theory has a long history [4] with lots of modern developments [5] that cannot be duly reviewed here. As part of the progress, a conceptual outline of nonperturbative QCD in the LF frame was achieved [6] using the similarity renormalization group procedure [7, 8]. A confining logarithmic potential of order α in quark-antiquark sector has been discovered [9] and studied in heavy quarkonia [10, 11] and other systems [12, 13], but not using RGPEP [14]. Besides the work that led to Refs. [1, 2], distinct RGPEP applications in scattering theory [15] and gluonium [16] are relatively new. But there is a lot of work to do in LF QCD before it can be widely accepted as a viable alternative to lattice QCD [17]. The AdS/CFT method [18, 19] has not been connected with RGPEP in QCD yet.

2   Bottomonium masses as example

In the crudest version, H_λ is calculated using RGPEP to first order in α_{λ_0} in one flavor QCD, and $H_{\lambda_0 Q\bar{Q}}$ is then evaluated to the same order, using the ansatz μ for gluons in the effective quark-antiquark-gluon sector (see next section). The resulting eigenvalue problem does not depend on the ansatz μ and takes a form that satisfies requirements of rotational symmetry despite

that the LF reference frame distinguishes z-axis. For example, the eigenstates with quantum numbers of Υ are described by a 2×2 matrix wave function $\phi(\vec{k}) = \vec{b}(\vec{k})\vec{\sigma}$, where ($\vec{s}$ is the polarization three-vector that determines the polarization state of the whole quarkonium in motion with arbitrary velocity)

$$b^m(\vec{k}) = \left[\delta^{mn} \frac{S(k)}{k} + \frac{1}{\sqrt{2}} \left(\delta^{mn} - 3\frac{k^m k^n}{k^2} \right) \frac{D(k)}{k} \right] s^n . \tag{1}$$

The matrix ϕ enters into the definition of a relativistic quantum state of a quarkonium with definite momentum and mass M,

$$|M, P^+, P^\perp, \vec{s}\rangle = \int [ij] \, (2\pi)^3 P^+ \delta^3(P - p_i - p_j) \, \chi_i^\dagger \, \phi(\vec{k}_{ij}) \chi_j \, b_{\lambda_0 i}^\dagger d_{\lambda_0 j}^\dagger |0\rangle , \tag{2}$$

where i and j denote flavor, momentum, and spin quantum numbers (colors are combined to 0) of scale-dependent effective quarks that are created by operators $b_{\lambda_0 i}^\dagger$ and $d_{\lambda_0 j}^\dagger$ from the LF QCD vacuum. These LF operators depend on the scale λ (in ratio to Λ_{QCD} in the RGPEP scheme, and the quark mass). The relative momentum three-vector $\vec{k}_{ij}$ is defined using LF kinetic momentum variables and the quark mass corresponding to the scale λ_0. The eigenvalue equation satisfied by the S and D wave functions is written in terms of dimensionless momentum variables

$$\vec{p} = \vec{k}_{ij}/k_B , \tag{3}$$

where k_B denotes the strong Bohr momentum, $\alpha_{\lambda_0} m_{\lambda_0}/2$ (subscript λ_0 will be dropped from now on). Using $p = |\vec{p}|$, the radial equation can be written as

$$\left[\begin{array}{cc} h_{osc} & 0 \\ 0 & h_{osc} + k_p \frac{6}{p^2} \end{array} \right] \left[\begin{array}{c} S(p) \\ D(p) \end{array} \right] = \int_0^\infty dk \, f \frac{2pk}{\pi} \left[\begin{array}{cc} \mathcal{W}_{ss} & \mathcal{W}_{sd} \\ \mathcal{W}_{ds} & \mathcal{W}_{dd} \end{array} \right] \left[\begin{array}{c} S(k) \\ D(k) \end{array} \right] \tag{4}$$

with

$$h_{osc} = p^2 - k_p \partial_p^2 - x , \tag{5}$$

$$k_p = \frac{9}{128\sqrt{2\pi}} \left(\frac{\lambda_0^2}{\alpha \, m^2} \right)^3 , \tag{6}$$

while the quarkonium mass eigenvalue is given by the eigenvalue x through

$$M = 2m \sqrt{1 + x \left(\frac{2}{3}\alpha \right)^2} . \tag{7}$$

Note that the eigenvalue is not energy in any specific frame of reference but the mass itself. The functions $\mathcal{W}_{ss}$, $\mathcal{W}_{sd}$, $\mathcal{W}_{ds}$, $\mathcal{W}_{dd}$ are given in the literature [2].

There is no quantitative trace of the gluon mass ansatz in this result. But there is a qualitatively new element in the form of a harmonic oscillator correction to the strong Coulomb potential (with LF Breit-Fermi terms).

Another qualitatively new element, a result of using RGPEP, is the form factor

$$ f = \exp\left\{ -\left[\frac{\mathcal{M}^2(p) - \mathcal{M}^2(k)}{\lambda_0^2} \right]^2 \right\}, \qquad (8) $$

where $\mathcal{M}$ denotes an invariant mass of a pair of free quarks. The form factor tempers the spin-dependent gluon exchange interaction. In particular, it regulates otherwise ultraviolet-divergent three-dimensional delta functions (in the position space formally associated with the momentum space of $\vec{k}$ via the Fourier transform), which are present in the functions $\mathcal{W}$ due to the relativistic spin effects.

One solves the eigenvalue equations for $b\bar{b}$ bound states, such as eq.4, assuming that α is given by the RGPEP evolution from the known value at $\lambda = M_Z$ down to λ_0. If $\alpha_{M_Z} = 0.12$, the lowest order RGPEP evolution of α_λ in QCD with only one flavor [20] produces $\alpha \sim 0.326$ at $\lambda_0 \sim 3.7$ GeV (about 30% smaller value is generated for 6 or 5 flavors). Less is known about the RGPEP evolution and value of the b-quark mass, m_b. Tab.1 shows masses of $b\bar{b}$ quarkonia obtained [2] when α and m_b are adjusted to reproduce masses of $\chi_1(1P)$ and $\chi_1(2P)$ at $\lambda_0 = 3697.67$ MeV.

If the RGPEP calculation of H_λ and subsequent reduction to $H_{\lambda Q\bar{Q}}$ were exact, there should be no dependence of the spectrum on λ. Once α and m_b are adjusted to observables at one scale, they evolve in some exact way, including the formation of bound states. But in this crudely simplified version of LF QCD, the RGPEP procedure is limited to order α and $H_{\lambda Q\bar{Q}}$ is finessed using an ansatz for the gluon mass gap μ. Therefore, one cannot change λ considerably using equations limited to order α and there is only a hope that in some small range of values of λ_0 the equations have a chance to work once the coupling constant and quark mass are given their right relativistic values [6, 22].

There is a characteristic pattern visible in the fourth column in tab.1: the greater the difference between a mass eigenvalue and the masses of quarkonia in the middle of the table, used to choose α and m_b, the greater the discrepancy

Table 1: Example of calculated masses (MeV) for $b\bar{b}$ states. The corresponding coupling constant and quark mass are $\alpha = 0.32595$ and $m_b = 4856.92$ MeV.

meson	theory	experiment [21]			difference
$\Upsilon 10860$	10725	10865	± 8		-140
$\Upsilon 10580$	10464	10579.4	± 1.2		-116
$\Upsilon 3S$	10382	10355.2	± 0.5		27
$\chi_2 2P$	10276	10268.65	± 0.22	± 0.50	7
$\chi_1 2P$	10256	10255.46	± 0.22	± 0.50	0
$\chi_0 2P$	10226	10232.5	± 0.4	± 0.5	-6
$\Upsilon 2S$	10012	10023.26	± 0.31		-11
$\chi_2 1P$	9912	9912.21	± 0.26	± 0.31	-1
$\chi_1 1P$	9893	9892.78	± 0.26	± 0.31	0
$\chi_0 1P$	9865	9859.44	± 0.42	± 0.31	5
$\Upsilon 1S$	9551	9460.30	± 0.26		91
$\eta_b 1S$	9510	9300	± 20	± 20	210

between the crudely approximated theory and experiment. This should be expected. The most strongly bound states are sensitive to deviations of the effective potential from the Coulomb shape. For example, interactions order g^4 (or α^2), introduce δ-functions that are absent here because of the limitation to terms order g^2 in the RGPEP and two-quark reduction. Analogous 4th order δs and other singular corrections are known in QED. Here such terms should have much larger effect because the coupling constant is about 30 times larger then in QED (they have to be treated nonperturbatively). The least strongly bound states, those with largest masses, should not be described well without proper inclusion of gluons. The mass ansatz should fail to render interactions that are associated with gluons producing a linear potential at distances much larger than the strong Bohr radius.

3 Mass ansatz as a computational tool in QCD

The mass ansatz for virtual effective particles in Fock components that contain more such particles than just two in mesons, or three in baryons, deserves a comment for several reasons. One reason is that the ansatz may help improve the calculations for heavy quarkonia. Another reason is that it may lead to a possibility of calculating properties of baryons built from heavy quarks. The

third reason is that it may help in crossing the barrier that separates all small-coupling expansions in QCD from entering the region of quark masses much smaller than Λ_{QCD}.

The first two reasons concern the difficulty that precise numerical solutions of eigenvalue problems with coupled two-, three-, and more-particle sectors are hard to obtain. In this respect, the mass ansatz appears a candidate to mimic what happens in an infinite tower of the Fock components built from effective particles. Since the RGPEP form factors limit momentum transfers by λ, the spread of probability to sectors with many effective particles corresponding to the scale λ is tamed. But these sectors do influence the dynamics of the dominant sectors and some ansatz appears inevitable. The question is how to make it self-consistently. The basic idea is to drop all sectors above the highest included (in the sense of number of the effective particles), put in instead a mass ansatz in the highest Fock component, and see what happens in the dynamics of lower components. The next step is to increase the maximal number of effective particles by one and see if the same type of ansatz is producing the same answers. A small coupling expansion may constrain the options sufficiently for finding good candidates for suitable mass terms in the highest components.

The third reason that concerns light quarks is most speculative. It involves chiral symmetry, or rather the mechanism of its breaking. In LF Hamiltonian of QCD with small λ, there may exist finite terms that violate chiral symmetry and do not vanish in the limit of quark mass approaching zero [6]. At the same time, the LF vacuum state remains simple due to cutoffs imposed on the particle momentum components along the front. The question is how to find those terms in practice. LF power counting limits the structure of allowed terms but so far insufficiently for anybody to tackle the issue, even though the stakes are high.

From this point of view, the following observation is of interest. Consider a colorless state built from two effective gluons. They attract each other. Consider then that one of these gluons turns into a pair of quarks. These quarks are in an octet state and instead of attracting they repel each other. In perturbation theory, if the number of quark flavors is not too high, this is not dangerous and gluonic interactions sustain asymptotic freedom, generating infrared slavery. However, beyond perturbation theory, a pointlike creation of

a pair of quarks that repel each other by violent potentials may lead to an explosive behavior. Such behavior is entirely absent in QED because electrons attract anti-electrons and this effect slows down the growth of a pair, instead of accelerating it.

To be more specific, consider eq.4, in which the s-wave potential $\mathcal{W}_{ss}$ contains a term [2] that would be a δ-function if the form factor f did not smooth it out. This term is attractive in Υ. It is repulsive instead in the color octet states. When λ is comparable to the heavy quark mass, which means that α is small, the smallness of the coupling constant and form factor width produce an interaction that cannot compete significantly with the size of the quark mass. This is visible in tab.1. However, if the quarks are light, it is entirely unclear what will happen.

The situation is different than in the case of analogy between a gluonium and a helium atom with one doubly charged electron discussed in Ref. [22]. Here, two particles with the same charge are suddenly put on top of each other and large potential energy is created.

In the case of light quarks, the large terms are smoothed out by the RGPEP form factors f but their strength may be comparable with Λ_{QCD}. The central point is that in order to find out what happens due the explosive nature of color dynamics of effective particles beyond perturbation theory, one has to separate some sectors from a presumably decreasing but in principle infinite chain of them. This is what the mass ansatz facilitates. Thus, it opens a way to investigate interactions in the effective Hamiltonians that may be calculated perturbatively in RGPEP and then diagonalized nonperturbatively using computers. This way one can find out if the effective interactions may in principle be responsible for emergence of the constituent quark masses for quarks u, d, and s.

4   Conclusion

Tab.1 shows that the chance that LF Hamiltonian approach to QCD may apply in phenomenology of heavy quarkonia is not hopelessly small. Since the coupling constant one needs is order 1/3, and one may need even a smaller coupling constant when 4th order RGPEP is used, the approach stands ready for a more extended scrutiny. The reason it deserves to be checked is that it appears now to indicate a possibility that a single formulation of the entire

theory with quark masses much greater than Λ_{QCD} is conceivable with no need to combine different formulations for including information concerning different scales.

The harmonic potential finessed in the quark-antiquark sector using an ansatz for the gluon mass gap, leads to the eigenvalues M^2 that are proportional to the angular momentum of relative motion of quarks, like in the Regge trajectories. It is found that the oscillator frequencies are on the order of one inverse fermi, and the oscillator potential grows as the relative distance squared in fermis with a coefficient given by the quark mass. So, presumably, for states with masses greater than order 1 GeV above the ground states, the probability of emission of effective gluons increases and then formation of strings of gluons is favored if the gluons also have some oscillator force acting among them. For quantum gluons to form a string, each pair of the neighboring gluons must be held together stronger than by a linear potential. If it is capable of generating such effects, the LF Hamiltonian approach could thus lead to a quantum theory of the gluon string in QCD without ever introducing a nontrivial vacuum.

But the concept of mass ansatz in Fock sectors with one more effective particle than the maximal number treated nonperturbatively, is probably most interesting as a tool for finding out what happens when one attempts to solve eigenvalue equations for Hamiltonians that are evaluated using perturbative RGPEP in the case of canonical QCD with quark masses smaller than Λ_{QCD}. The idea discussed here is that effective particles in non-singlet color configurations may experience explosive potentials in the form of smoothed δ-functions that may cause effects order Λ_{QCD} per constituent. The ansatz for quark and gluon masses inserted in the highest sectors thus opens a possibility to generate concrete forms of such terms and to study them nonperturbatively. Nothing is known yet about what may come out from such studies.

5   Acknowledgements

References

1. S. D. Głazek, Phys. Rev. D **69**, 065002 (2004).

2. S. D. Głazek, J. Młynik, Phys. Rev. D **74**, 105015 (2006).

3. S. D. Głazek, in: Continuous Advances in QCD 2006 (ed. M. Peloso and M. Shifman, Minneapolis, May 2006), 496 (World Scientific, 2007).

4. S. J. Brodsky et al., Phys. Rept. **301**, 299 (1998).

5. http://www.slac.stanford.edu/xorg/ilcac .

6. K. G. Wilson et al., Phys. Rev. D **49**, 6720 (1994).

7. S. D. Głazek, K. G. Wilson, Phys. Rev. D **48**, 5863 (1993).

8. S. D. Głazek, K. G. Wilson, Phys. Rev. D **49**, 4214 (1994).

9. R. J. Perry, in: Hadron Physics 94 (ed. V. E. Herscovitz et al., Gramado, April 1994), 120 (World Scientific, 1995); hep-th/9407056v2.

10. M. Brisudova, R. Perry, Phys. Rev. D **54**, 1831 (1996).

11. M. M. Brisudova et al., Phys. Rev. Lett. **78**, 1227 (1997).

12. B. H. Allen, R. J. Perry, Phys. Rev. D **62**, 025005 (2000).

13. R. D. Kylin, Ph.D. Thesis, The Ohio State University, 2001.

14. M. M. Brisudova, Mod. Phys. Lett. A **17**, 59 (2002).

15. M. Więckowski, hep-th/0511148.

16. T. Masłowski, PhD Thesis, University of Warsaw, 2005.

17. K. G. Wilson, Nucl. Phys. (Proc. Suppl.) **140**, 3 (2005).

18. S. J. Brodsky, G. F. de Teramond, Phys. Rev. Lett. **96**, 201601 (2006).

19. S. J. Brodsky et al., Phys. Rev. D **75**, 014003 (2007).

20. S. D. Głazek, Phys. Rev. D **63**, 116006 (2001).

21. Particle Data Group website, pdg.lbl.gov/2007/tables/mxxx.pdf

22. S. D. Głazek, K. G. Wilson, Phys. Rev. D **57**, 3558 (1998).